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1987 J. Phys. A: Math. Gen. 20 L993

(http://iopscience.iop.org/0305-4470/20/15/011)

J. Phys. A: Math. Gen. **20** (1987) L993-L996. Printed in the UK

LETTER TO THE EDITOR

Dynamics of the Potts model on a fractal lattice

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Received 4 August 1987

Abstract. The dynamics of the q-state Potts model on a fractal lattice is studied using Monte Carlo simulations. The Glauber dynamics is used leading to an effective temperature-dependent critical exponent of the form $z = AK + B$ implying the breakdown of conventional dynamic scaling. The value of A is shown to be independent of q, within the error bars.

Recently a number of studies on the Ising model at percolation threshold [1-3] have shown that the conventional dynamic scaling breaks down at low temperatures, i.e. the dynamic critical exponent becomes temperature dependent and diverges at $T = 0$. These theoretical works were motivated by a recent neutron scattering experiment [4] on the dilute, two-dimensional Ising antiferromagnet $Rb_2(Mg_{0.41}Co_{0.59})F_4$ near the percolation threshold. By fitting the measured relaxation time τ to a power law $\tau \sim \xi^z$, predicted by conventional dynamic scaling, it was found [4] that $z = 2.4 \pm 0.1$, which is an exceptionally large value. This result led to the suggestion that the dynamics of the system at low temperatures was governed by a different type of dynamics. The underlying cause for the change in the spin dynamics was suggested [1] to be in the fractal nature of the percolating cluster. At low temperatures the spin dynamics is determined by thermal activation over energy barriers. In contrast to the power law behaviour found in Euclidean lattices, the energy barrier scales logarithmically with the cluster size L in fractal networks.

This novel dynamical behaviour was subsequently found in the two-dimensional Ising [5] and Potts [6, 7] models at the percolation threshold, using Monte Carlo [5, 6] and renormalisation group [7] methods. Dynamics in hierarchical Ising systems [8] and Monte Carlo studies of the Ising model in fractal lattices [9, 10] have also shown this singular dynamics.

In studying the dynamics of the Potts model on a fractal network, one may ask how the effective dynamic exponent depends on the number of states. We try to answer this question by analysing the low-temperature behaviour of the relaxation time for the Potts model in a fractal lattice. One advantage of working with fractal lattices is that the behaviour of the relaxation time τ is expected to be the same, in the limit $L \ll \xi$, down to the smallest size (as opposed to percolating clusters), allowing us therefore to work with small-size systems [1, 9]. In addition, because of the fractal nature of the incipient infinite percolating cluster, we expect that the results can also be applied to the two-dimensional Potts model at the percolation threshold.

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0305-4470/87/150993+04$02.50

The system under investigation here is described by the Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \delta_{\sigma_i \sigma_j} \tag{1}$$

where $\sigma_i = 0, 1, \ldots, q-1$, specifies the spin state at site i, and the summation is over nearest neighbours only. The spins are placed on every site of a finitely ramified fractal lattice (see figure 1). The physical quantity studied in the simulation is the time-dependent magnetisation $M(t)$ given by

$$M(t) = \frac{1}{q-1}\left(q\left\langle \frac{1}{N} \sum_i \delta_{0,\sigma_i(t)} \right\rangle - 1 \right) \tag{2}$$

where $\sigma_i(t)$ is the state of the ith spin at time t. The averages are taken over a large number of lattices with N spins each.

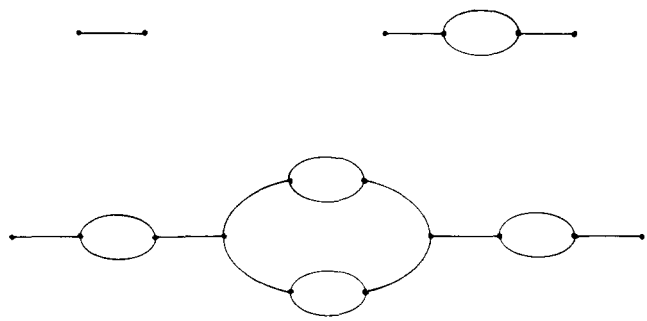

Figure 1. The first three stages of the fractal lattice used in the simulation.

Assuming that dynamic scaling holds, the magnetisation, at low temperatures ($T_c = 0$), obeys the scaling relation:

$$M(t, K, L) = M(t/\Omega, K', L/b) \tag{3}$$

where b and Ω are the length and time rescaling factors, respectively. The coupling K' obtained by decimation (with $b = 3$) is given, for the fractal lattice shown in figure 1, by

$$\exp(K') = \frac{\exp(2K) + q - 1}{\exp(K) + q - 2}. \tag{4}$$

A direct consequence of (3) is the following scaling form for the relaxation time:

$$\tau(L, K) = \Omega(b, K)\tau(L/b, K'). \tag{5}$$

The conventional scaling dynamics predicts that $\Omega = b^z$, where z, the dynamic critical exponent, is a constant. For fractal networks the dynamic exponent z is temperature dependent, showing therefore a breakdown of the conventional dynamic scaling.

Monte Carlo simulations were performed with the Hamiltonian (1) to study the time dependence of the magnetisation in lattices of 12 and 44 spins and linear sizes (L) of 9 and 27, respectively. We have taken the starting configuration to be one in which all spins are in the same state $\sigma_i = 0$ and let the system evolve towards equilibrium under Glauber dynamics.

The spins are flipped with transition probability $p = \exp(-\Delta E/k_B T_c)/[1+\exp(-\Delta E/k_B T_c)]$ where ΔE denotes the change in energy resulting from an update of a randomly selected spin. In all cases the simulation was performed at low temperatures as close as possible to $T_c = 0$.

In figures 2 and 3 we present the results of the simulation. Using the same procedure as [10] the magnetisation per site in each lattice was measured as a function of time and averaged over many samples (10^5-10^6). The values of the magnetisations were made to match by shifting the time scale for the small lattice (at $T' = 1/K'$) by a factor

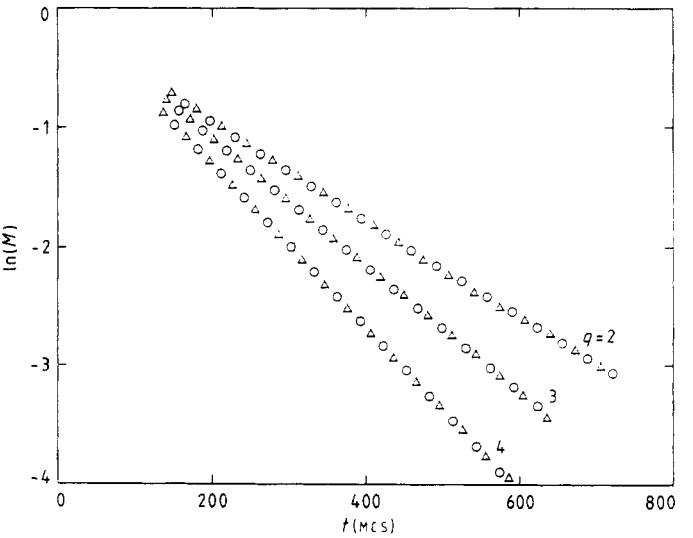

Figure 2. Typical data collapse found in the simulation done with lattices having 12 spins (○) and 44 spins (△). Measurements were performed at $K = 1.8$. The time rescaling factors used are $\Omega(2) = 16.4$, $\Omega(3) = 15.6$ and $\Omega(4) = 15.1$.

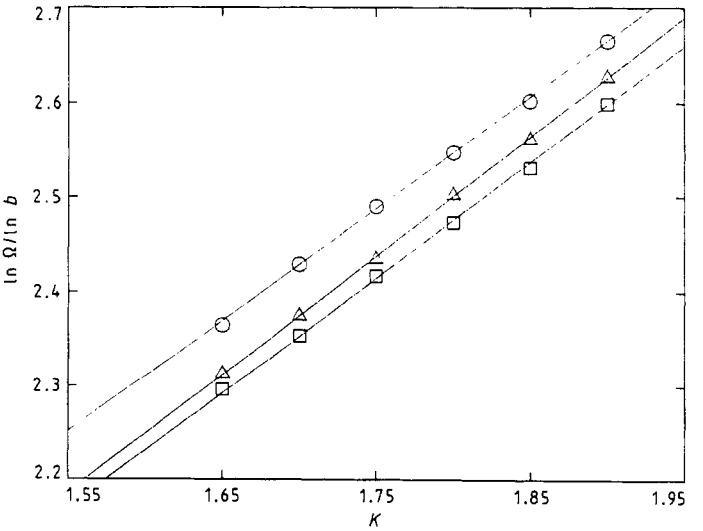

Figure 3. Temperature dependence of the relaxation time $\Omega(L, K)$ for $q = 2$ (○), 3 (△) and 4 (□). The full lines are the linear best fits.

Ω (see figure 2). The renormalised temperature for the small lattice was obtained using (4). The value for $\Omega(L, K)$ at a given temperature was found by taking an average over several matching points (15–30). The collapse of the data (for all values of q) in a large interval of time, as shown in figure 2, reflects the dynamic scaling of the magnetisation as predicted by (3).

The resulting relaxation times present a clear temperature dependence of the form $z(q) = \log_b \Omega = AK + B$. An additional interesting feature is the independence of the slopes with respect to q. A linear best fit on the data of figure 3 gives the following results for the slopes: $A(2) = 1.18 \pm 0.05$, $A(3) = 1.26 \pm 0.04$ and $A(4) = 1.22 \pm 0.03$; and for the intercepts: $B(2) = 0.42 \pm 0.10$, $B(3) = 0.23 \pm 0.07$ and $B(4) = 0.27 \pm 0.06$. The value for the slope for $q = 2$ seems to be a little higher than that obtained for the Ising model on the same lattice [10].

These results are in fact in contradiction with those recently obtained for the Potts model in a square lattice at the percolation threshold, using Monte Carlo simulations [6] and real space renormalisation group methods [7]. These studies predict a relatively strong q dependence for the slopes. One should emphasise that the present results, although obtained for small lattices, are expected to be independent of the lattice size since the behaviour of the relaxation time should be the same in a fractal lattice down to the smallest size. Additional work on the dynamics of the Potts model using different fractal geometries will be presented elsewhere.

I thank Professor P O Löwdin for his continuous encouragement and Professor R Chow, Director of Computer Information Sciences, for provision of additional computer time.

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